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Asymptotic Reduced-Dimensional Steering Strategies for CMG Singularity-Free Control

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Abstract

The geometric singularity problem is one of principal difficulty when using single-gimbal control moment gyros as spacecraft attitude control devices. To overcome singularity, new steering logics are suggested in this paper which results in a reduction in the difficulty of generating gimbal rates around a singular state. One of the suggested steering laws presented is the reduced dimensional singular value decomposition steering law, which adopts the singular value decomposition in reduced dimensional forms. Two other steering laws make use of the least square method in reduced dimensional forms. All of the suggested steering laws have been generated for the compensation of the torque insufficiency. These logics are verified mathematically and simulations at a singular condition and non-singular condition are performed to see how well they work.

Keywords: CMG, Steering law, Singularity-free control, Switching parameter

1. Introduction

Single Gimbal Control Moment Gyros (SGCMGs) have a significant advantage over other attitude control devices such as thrusters or reaction wheels in terms of torque amplification. Therefore, expectations have been that SGCMGs will become increasingly necessary for agile spacecraft. However, despite such an advantage, it is difficult to use SGCMGs as spacecraft attitude control devices because of the geometric singularity phenomenon, which is a specific arrangement of gimbals leading to a loss of controllability of 3-axes. Singular states preclude SGCMGs from generating torque in a certain direction, called the singular direction, and lead to loss of three-axis control of the spacecraft. Many studies and extensive development have been sought to solve this singular problem. Margulies and Auburn (1978) established the fundamentals of the geometric analysis for singular conditions. Cornick (1979) and Bedrossian et al (1990) suggested the use of null motion to avoid singularity. Nakamura and Hanafusa (1986) suggested the singularity robust inverse method. Vadali et al (1990), Oh and Vadali (1994) and Wie et al (2001) have extended the application of this method. Ford and Hall (2000) make use of singular value decomposition to compute a pseudoinverse that prevents large gimbal rate commands. However, these methods allow torque errors to avoid the singularity or command large gimbal rates around the singularity. For these reasons, a new method that decreases torque errors while generating gimbal rates without an abrupt change is needed.

In this paper, three new types of steering logics are suggested. These logics operate gimbal rates robustly in order to obtain output torque by means of asymptotic reduced-dimensional fashions rather than by

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avoiding the singularity directly. By observing the Jacobian matrix and using the singular value decomposition, the Reduced Dimensional Singular Value Decomposition (RDSVD) steering law is designed by eliminating one of singular values causing singularity during the process of reducing the dimension of the Jacobian matrix. In this steering logic, the insufficient output torques which lead to torque errors are filled by compensation. However, the chattering phenomena of gimbal rates occur during the process of compensation. To prevent these chattering phenomena, newly defined parameters that play on/off roles are used to help produce gimbal rates without chattering.

As a different approach, two different steering laws are invented by means of the least square method in which a form of reduced dimensional Jacobian matrix is used. These reduced dimensional matrices are composed of singularity-free forms. They are called Reduced Dimensional Least Square (RDLS) steering laws. These suggested laws also require compensation on the insufficient output torques that occur when the Jacobian matrix in the reduced dimensional forms are adopted. For the compensation, a series of gimbal rate commands are generated in an asymptotic fashion in regular sequence.

To verify the methods suggested in this paper, some illustrative simulations are performed on singular sate and non-singular state conditions.

2. Spacecraft system with four SGCMGs

A spacecraft with four single-gimbal CMGs mounted in the pyramid configuration is considered as shown in Fig. 1. The moment of inertia of the gimbals is assumed negligible compared to that of the spacecraft.

The total angular momentum vector h of the SGCMGs cluster can be described in terms of the sum of the i-th wheel axial angular momentum h_i as

$$h = \sum_{i}^{4} h_i(\sigma_i) \tag{1}$$

where h_i is the function of gimbal angle σ_i. The time derivative of h is the output torque, and can be written as

$$\dot{h} = D\dot{\sigma} \tag{2}$$

where the gimbal rate $\dot{\sigma}$ is denoted as $\dot{\sigma} \equiv [\,\dot{\sigma}_1\ \dot{\sigma}_2\ \dot{\sigma}_3\ \dot{\sigma}_4]^T$. The matrix D is a 3 × 4 Jacobian matrix shown by

$$D \equiv [\,d_1\ d_2\ d_3\ d_4\,] \equiv [\,\frac{dh_1}{d\sigma_1}\ \frac{dh_2}{d\sigma_2}\ \frac{dh_3}{d\sigma_3}\ \frac{dh_4}{d\sigma_4}] \tag{3}$$

where the 3×1 column vector d_i represents the output torque produced by the rotation of the i-th gimbal.

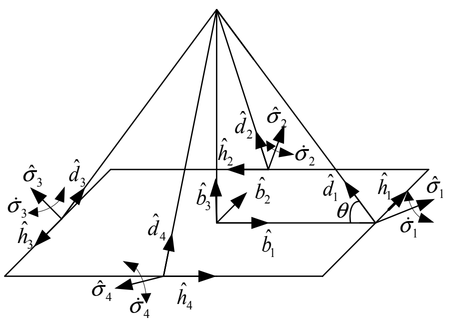


Fig. 1. A Pyramid Configuration of four SGCMGs.

3. Steering laws and singularities

To produce the required torque L_r from Eq. (2), since the Jacobian matrix D is 3×4, a control solution for gimbal rates can be selected using a pseudo inverse D^+ shown as

$$\dot{\sigma} = D^+ L_r = D^T (DD^T)^{-1} L_r \tag{4}$$

However, when all of the output torque vectors d_i are located at a coplanar level, the rank of D then becomes less than three. This is called the singularity. Methods to overcome this singularity have been developed, but they allow for torque errors and an abrupt change of gimbal rates in the vicinity of singular states. Thus, a new steering strategy that reduces the torque errors and difficulties in generating gimbal rates around singularity is needed. To this end, an attempt is made to generate a series of gimbal rate commands in an asymptotic fashion by adopting the reduced dimensional form of matrix D. Three types of reduced dimensional steering laws are suggested: Reduced Dimensional Singular Value Decomposition (RDSVD), and Reduced Dimensional Least Square (RDLS)-1 and -2.

3.1 Reduced dimensional singular value decomposition steering law (RDSVD)

The matrix D can be decomposed by singular value decomposition into the product of three special matrices represented as

$$D = USV^T \tag{5}$$

where U is a 3×3 unitary matrix, S is a 3×4 singular value matrix, and V is a 4×4 unitary matrix. The last column of S is always all zero. Thus, S can be replaced by the truncated matrix S_t with singular values s_1, s_2, s_3, which are obtained by discarding the last column of S. The matrix D can thus be represented by the truncated matrices as

$$D = US_tV_t^T \tag{6}$$

The matrix V_t is the matrix in which the last column is truncated from V. The pseudo inverse of D can then be represented as

$$D^+ = D^T(DD^T)^{-1} = V_tS_t^{-1}U^T \tag{7}$$

where $S_t^{-1} = diag\{\frac{1}{s_1}, \frac{1}{s_2}, \frac{1}{s_3}\}$ (8)

When the matrix D becomes singular, the singular value S_3 approaches zero, and S_t^{-1} moves toward infinity. To avoid this harsh condition of singularity, modified reduced dimensional matrices are considered. These modified matrices are made by eliminating the singular value s_3 from singular value matrix and, to rearrange the rest of singular values s_1 and s_2, U and V matrices are divided into the corresponding dimensional forms.

$$\begin{aligned} &U_{t1} \equiv [u_2\ u_3],\quad U_{t2} \equiv [u_1\ u_3],\quad U_{t3} \equiv [u_1\ u_2] \\ &V_{t1} \equiv [v_2\ v_3],\quad V_{t2} \equiv [v_1\ v_3],\quad V_{t3} \equiv [v_1\ v_2] \\ &S_{tt} \equiv \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \end{aligned} \tag{9}$$

where U_{ti} is a 3×2 matrix, V_{ti} is a 4×2 matrix, and S_{tt} is a 2×2 singular value matrix . Then, a modified inversion of the matrix D is considered as

$$D^* = \frac{1}{2}\sum_{i=1}^{3}(V_{ti}S_{tt}^{-1}U_{ti}^T) = V_tS_t^*U^T \tag{10}$$

$$S_t^* \equiv diag\{\frac{1}{s_1}, \frac{1}{2}(\frac{1}{s_1}+\frac{1}{s_2}), \frac{1}{s_2}\} \tag{11}$$

The elements of S_t^{-1} in Eq. (7) are rearranged with respect to the non-zero singular values of s_1 and s_2. The gimbal rate command is then chosen as

$$\dot{\sigma}_{s1} = (V_tS_t^*U^T)L_r \equiv D_t^*L_r \tag{12}$$

The gimbal rate command obtained above is, however, not sufficient to produce the required torque exactly. For this reason, gimbal rate compensation is needed and an iterative asymptotic approach is attempted as follows. Let L_{o1} be the output torque generated by the gimbal rate command $\dot{\sigma}_{s1}$ as

$$\begin{aligned} D\dot{\sigma}_{s1} &= DD_t^*L_r \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{s_1+s_2}{2s_1} & 0 \\ 0 & 0 & \frac{s_3}{s_2} \end{bmatrix} L_r = L_{o1} \end{aligned} \tag{13}$$

The output torque obtained with gimbal rate $\dot{\sigma}_{s1}$ is normally different from the required torque because Jaconian matrix D is changed into the reduced dimensional matrices. In this case, to compensate on output torque, another gimbal rate is needed and can be obtained with the use of torque difference $L_r - L_{o1}$ as with Eq. (14). This is similar process obtaining gimbal rate $\dot{\sigma}_{s1}$ except for the use of torque difference instead of required torque.

$$\dot{\sigma}_{s2} = D_t^*(L_r - L_{o1}) \tag{14}$$

Then, the output torque generated by the gimbal rate command $\dot{\sigma}_{s2}$ is obtained in the same way.

$$\begin{aligned} D\dot{\sigma}_{s2} &= DD_t^*(L_r - L_{o1}) \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{s_1+s_2}{2s_1}(1-\frac{s_1+s_2}{2s_1}) & 0 \\ 0 & 0 & \frac{s_3}{s_2}(1-\frac{s_3}{s_2}) \end{bmatrix} L_r = L_{o2} \end{aligned} \tag{15}$$

Similarly, the corresponding output torque for the next generation of gimbal rate commands, represented by $\dot{\sigma}_{s3}$, $\dot{\sigma}_{s4}$ are obtained respectively as

$$\dot{\sigma}_{s3} = D_t^*(L_r - L_{o1} - L_{o2}) \tag{16}$$

$$D\dot{\sigma}_{s3} = DD_t^*(L_r - L_{o1} - L_{o2}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{s_1+s_2}{2s_1}(1-\frac{s_1+s_2}{2s_1})^2 & 0 \\ 0 & 0 & \frac{s_3}{s_2}(1-\frac{s_3}{s_2})^2 \end{bmatrix} L_r = L_{o3} \tag{17}$$

$$\dot{\sigma}_{s4} = D_t^*(L_r - L_{o1} - L_{o2} - L_{o3}) \tag{18}$$

$$D\dot{\sigma}_{s4} = DD_t^*(L_r - L_{o1} - L_{o2} - L_{o3}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{s_1+s_2}{2s_1}(1-\frac{s_1+s_2}{2s_1})^3 & 0 \\ 0 & 0 & \frac{s_3}{s_2}(1-\frac{s_3}{s_2})^3 \end{bmatrix} L_r = L_{o4} \tag{19}$$

Then, the gimbal rate command is obtained by combing all the gimbal rates as

$$\dot{\sigma} = \dot{\sigma}_{s1} + \dot{\sigma}_{s2} + \dot{\sigma}_{s3} + \dot{\sigma}_{s4} \tag{20}$$

Therefore, the output torque generated by the combined gimbal rate command is

$$D\dot{\sigma} = L_o = L_{o1} + L_{o2} + L_{o3} + L_{o4} \tag{21}$$

where $L_{o2} \equiv D\dot{\sigma}_{s2}$, $L_{o3} \equiv D\dot{\sigma}_{s3}$, and $L_{o4} \equiv D\dot{\sigma}_{s4}$

In Eq. (21), each sum of the compensated output torque is

$$\begin{aligned} L_o(1) &= L_r(1) \\ L_o(2) &= \left(\frac{s_1+s_2}{2s_1} + \frac{s_1+s_2}{2s_1}(\frac{s_1-s_2}{2s_1}) + \frac{s_1+s_2}{2s_1}(\frac{s_1-s_2}{2s_1})^2 + \frac{s_1+s_2}{2s_1}(\frac{s_1-s_2}{2s_1})^3 \right) L_r(2) \\ L_o(3) &= \left(\frac{s_3}{s_2} + \frac{s_3}{s_2}(\frac{s_2-s_3}{s_2}) + \frac{s_3}{s_2}(\frac{s_2-s_3}{s_2})^2 + \frac{s_3}{s_2}(\frac{s_2-s_3}{s_2})^3 \right) L_r(3) \end{aligned} \tag{22}$$

From Eq. (22) above, $L_o(2), L_o(3)$ are geometric progressions with the ratios $(s_1 - s_2)/2s_1$ and $(s_2 - s_3)/s_2$ respectively, and the sums of those geometric progressions are

$$L_0(2) = (1-(\frac{s_1-s_2}{2s_1})^n)L_r(2) \tag{23}$$

$$L_0(3) = (1-(\frac{s_2-s_3}{s_2})^n)L_r(3) \tag{24}$$

Here, the singular values s_1, s_2, s_3 have the relationship $s_1 \geq s_2 \geq s_3 \geq 0$. Further, values for required torques and output torques tend to merge as the number of compensations increases, as illustrated by Eqs. (25) and (26)

$$0 \leq \frac{s_1-s_2}{2s_1} \leq 1 \quad \rightarrow \quad \lim_{n\to\infty}(\frac{s_1-s_2}{2s_1})^n = 0 \tag{25}$$

$$0 \leq \frac{s_2-s_3}{s_2} \leq 1 \quad \rightarrow \quad \lim_{n\to\infty}(\frac{s_2-s_3}{s_2})^n = 0 \tag{26}$$

Equations (25) and (26) show that output torque converges on the required torque by the gimbal rate compensation method, if compensation is conducted continuously. Actually, singular value decomposition is computed just one time in every time step during simulations. However, there is no time consuming to affect real-time capability. Once singular value decomposition is computed, we get three matrices and there are left just simple calculation. In this approach, gimbal rate compensation is conducted only four times, that is *n*=4, and which is implemented compulsorily. Thus, torque differences still exist. But, such torque differences are very small respect to the required torque since compensation is already conducted four times. For example, which is shown in simulations in the later part of this paper, SR steering law also allows small amount of torque differences. Conducting compensation more than four times is also possible and achieves greater accuracy. However, it was found that the differences between required torque and output torques become very small after four rounds of compensation.

From the above Eq. (22), it is evident that the singular value s_3 becomes zero in singularity, which then causes the output torque $L_o(3)$ to be zero. That is, under the singular conditions, the output torque is

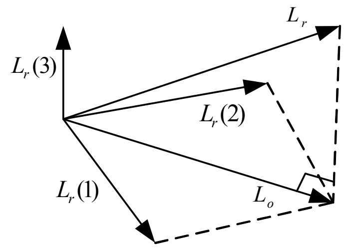


Fig. 2. Torque producing in a singular state.

produced in the plane made by $L_o(1)$ and $L_o(2)$, like the least square method in Fig. 2, if compensation is fully implemented. However, in the non-singular case, the output torque meets the required torque because of the methods of compensation.

In a singular condition, there exists a singular direction for which SGCMGs do not generate output torque. But, this suggested steering law compulsorily tries to produce output torque in that direction and such a condition makes output torque vectors of SGCMGs oscillate around a singular direction and a chattering phenomenon occurs. To prevent the chattering phenomenon, a switching parameter is newly defined. This parameter plays an on/off role much like a switch. That is, if a certain obtainable value goes below the assigned value, this parameter produces zero, if not, it produces one. This switching parameter is just made by a simple arithmetic. Considering a fraction, it will be zero if a numerator is zero and it will be one if a numerator is same with a denominator. So, switching parameter is based on such a thought and be represented in Eq. (27).

$$\alpha_b = \lambda_1 \left[(\lambda_1 + 1) + \lambda_2 \gamma^{-1} \right]^{-1} \tag{27}$$

where $b =$ assigned value

$\gamma = | \, b -$ obtained value$\, | - (b -$ obtained value$)$

$\lambda_1, \lambda_2 =$ constant

With this switching parameter, the gimbal rate is redesigned as

$$\dot{\sigma} = (K_\delta (1-\alpha_\varepsilon) + \alpha_\varepsilon)\dot{\sigma}_{s1} + \alpha_{0.1}\dot{\sigma}_{s2} + \alpha_{0.3}\dot{\sigma}_{s3} + \alpha_{0.5}\dot{\sigma}_{s4} \tag{28}$$

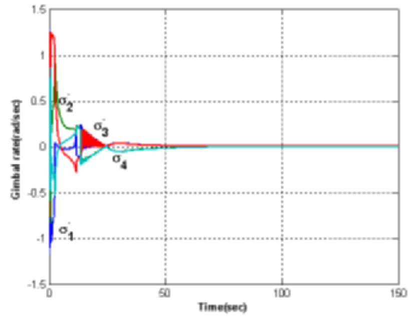
where $\varepsilon = \sum_{i=1}^{3} I_i \omega_i - 1.5$

$K_\delta =$ Constant

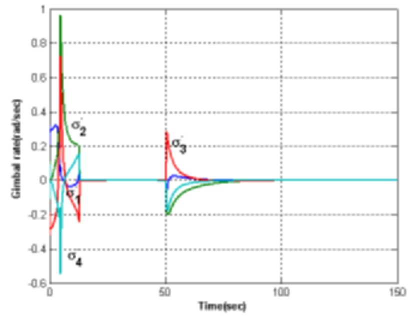
Then, Eq. (28) represents the actual gimbal rate command according to the value of a singularity measure. As a singularity measure, a determinant of DD^T is used. The constant values λ_1, λ_2 are set to 10^5 and 10^{-3} respectively, and K_δ is set to 10^{-2}.

Illustrative figures are shown below with a chattering phenomenon compared with the case of using and no-using switching parameter.

Above figure is a part of simulation 1 in this paper. Fig. (a) shows gimbal rate chattering phenomenon and Fig. (b) is the case of using switching parameter to eliminating a chattering of gimbal rate.



(a)



(b)

Fig. 3. A chattering phenomenon.

3.2. *Reduced dimensional least square steering law-1 (RDLS-1)*

Another singularity-free steering strategy is to select a pair of non-collinear output torque vectors among $\{d_1, d_2, d_3, d_4\}$, and then generate the required torque by the method of least-squares. For example, the Jacobian matrix D is divided into four 3×2 matrices, as in Eq. (29).

$$D_1 \equiv [d_1 \; d_2],\; D_2 \equiv [d_2 \; d_3],\; D_3 \equiv [d_3 \; d_4],\; D_4 \equiv [d_4 \; d_1] \tag{29}$$

Each of the four matrices in Eq. (29) is recomposed into singular value matrixes as in Eq. (30).

$$D_i = USV^T \ , i = 1, 2, 3, 4 \tag{30}$$

where U is a 3×3 unitary matrix, S is a 3×2 singular value matrix, and V is a 2×2 unitary matrix. The last row of S is always a row of zeroes. Thus, S can be replaced by the truncated matrix S_t with singular values s_1, s_2, which are obtained by discarding the last row of S, and the matrix U_t, which is also truncated from the last column of U. The matrix D_i can then be represented by the truncated matrices as

$$D_i = US_tV_t^T \tag{31}$$

Using the truncated matrices in Eq. (31), the gimbal rates for the first and second CMGs are then obtained from the least square method in the reduced dimensional form

$$\dot{\sigma}_{12} = (D_1^T D_1)^{-1} D_1^T L_r \tag{32}$$

where $\dot{\sigma}_{12}$ is the reduced dimensional gimbal rate command vector represented by $\dot{\sigma}_{12} \equiv [\,\dot{\sigma}_1 \ \dot{\sigma}_2]^T$. The output torque generated by the gimbal rate command $\dot{\sigma}_{12}$ is then

$$\begin{aligned} L_{o1} &= D_1 \dot{\sigma}_{12} \\ &= D_1 (D_1^T D_1)^{-1} D_1^T L_{r1} \\ &= U_t S_t V^T (V S_t^T U_t^T U_t S_t V^T)^{-1} V S_t^T U_t^T L_{r1} \\ &= U_t U_t^T L_{r1} \end{aligned} \tag{33}$$

In Eq. (33) above, $U_t U_t^T$ is a orthogonal projection matrix (Darald J. Hatfield, 2000) which projects the required torque to the plane made by the first and the second CMGs output torque, as illustrated in Fig. 4. Then, since L_{o1} is normally different from the required torque L_r, in order to reduce the difference, another gimbal rate command $\dot{\sigma}_{23}$ is generated as

$$\dot{\sigma}_{23} = (D_2^T D_2)^{-1} D_2^T (L_r - L_{o1}) \tag{34}$$

However, since it is still possible for a difference to exist between the output and the required torque, the gimbal rate commands $\dot{\sigma}_{34}$ and $\dot{\sigma}_{41}$ are obtained in a similar fashion consecutively. To combine the gimbal rates, the gimbal rate configuration matrices are used. These matrices play a role of assigning each gimbal rate to their corresponding positions. Finally, combining the gimbal rates result in

$$\dot{\sigma} = P_{12}\dot{\sigma}_{12} + P_{23}\dot{\sigma}_{23} + P_{34}\dot{\sigma}_{34} + P_{41}\dot{\sigma}_{41} \tag{35}$$

where the gimbal rate configuration matrices P_{12}, P_{23}, P_{34} and P_{41} are

$$\begin{aligned} P_{12} &\equiv \begin{bmatrix} 1000 \\ 0100 \end{bmatrix}^T, \ P_{23} \equiv \begin{bmatrix} 0100 \\ 0010 \end{bmatrix}^T, \\ P_{34} &\equiv \begin{bmatrix} 0010 \\ 0001 \end{bmatrix}^T, \ P_{41} \equiv \begin{bmatrix} 0001 \\ 1000 \end{bmatrix}^T \end{aligned} \tag{36}$$

In a non-singular state, the output torque generated by the combined gimbal rate will approach the required torque in Eq. (37), and finally converge after a few rounds of compensation.

$$\left\| L_{r,i} - L_{o,i} \right\| = \left\| L_{o,i+1} \right\| \cdot \sin\theta_i \quad , i = 1, 2, 3, 4, \cdots \tag{37}$$

Since each of four matrices in Eq. (29) has a different torque plane, a magnitude of the differences between the required torque and output torque $\left\| L_{ri} - L_{oi} \right\|$ gradually decrease to zero in a non-singular condition because the angle of θ_i is always $0 \le \theta_i < \frac{\pi}{2}$. This is well described in Fig. 4.

However, around the singularity, the output torque is produced only by the orthogonal projection matrix to the torque plane on which each SGCMG generates its capable torque. Although difference between the output torque L_o and the required torque L_r still exist, singular configurations can be removed by such a torque. That is, a singular SGCMG system can be reconfigured by using RDLS-1, and this difference is minimized by means of the least square method.

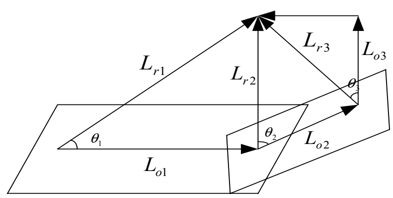


Fig. 4. Torque producing by RDLS-1.

3.3 Reduced Dimensional Least Square steering law-2 (RDLS-2)

Another least square steering strategy is to generate the required torque by operating the SGCMG independently, in other words by operating the output torque vectors $\{d_1, d_2, d_3, d_4\}$. This process is similar to the RDLS-1 except that the dimension is different from that of RDLS-1. At first, a least square method is performed on the first SGCMG resulting in

$$\dot{\sigma}_1 = (d_1^T d_1)^{-1} d_1^T L_r = \frac{d_1^T}{\|d_1\|^2} L_r \tag{38}$$

The output torque of the first SGCMG is then,

$$D\dot{\sigma}_1 = L_{o1} \tag{39}$$

Next, the asymptotic approach is applied, as shown previous section, in Eq. (40).

$$\dot{\sigma}_2 = \frac{d_2^T}{\|d_2\|^2}(L_r - L_{o1}) \tag{40}$$

The rest of the gimbal rates are individually obtained from the difference torque, and finally we obtain the combined gimbal rate command as

$$\dot{\sigma} = [\dot{\sigma}_1 \ \dot{\sigma}_2 \ \dot{\sigma}_3 \ \dot{\sigma}_4]^T \tag{41}$$

This steering law started from the same idea as that of RDLS-1. The output torque produced by each gimbal rate is an orthogonal projection torque of a required torque and goes through the same process as that in RDLS-1. In this steering law, the projection matrix in Eq. (33) plays to project the required torque to the line made by each gimbal rate. As with RDLS-1, there still exists some difference between the output torques and required torques around the singularities

4. Simulation Results

In order to verify the performance of the proposed steering laws, some illustrative example maneuvers are simulated. The physical model chosen is a rigid spacecraft with a cluster of four CMGs arranged in a pyramid configuration (Oh and Vadali, 1991). The simulation is conducted with a moment of inertia at $I \equiv \text{diag}\{86.215,\ 85.070,\ 113.565\}\text{Kg}\cdot\text{m}^2$ and SGCMG with an angular momentum capacity of 1.8 N·m·sec with a pyramid skew angle of 54.74 deg. A feedback control law for stabilizing to the target attitude (β_f , ω_f) is obtained by the Lyapunov approach as (Oh and Vadali, 1994)

$$D\dot{\sigma} = L_r \tag{42}$$

where L_r is the required torque defined as

$$L_r = K(\omega - \omega_f) - kG^T(\beta)\beta_f - \omega^\times I\omega - \omega^\times D\dot{\sigma} \tag{43}$$

where $\omega^\times = [0 - \omega_3\ \omega_2; \omega_3\ 0 - \omega_1; -\omega_2\ \omega_1\ 0]$

The gain is selected as K=diag{13.13, 13.04, 15.08}N·m·sec and k=diag{1.0, 1.0, 1.0}N·m. In simulation 1, a single axis maneuver is deliberately initiated from an elliptic singularity, as shown in Table 1, where the required torque is parallel to the singular direction. In simulation 2, a rest-to-rest maneuver is accomplished with the parameters, as shown in Table 2. The performance of the proposed steering laws is compared with that of the singularity robust (SR) steering law (Oh and Vadali, 1991).

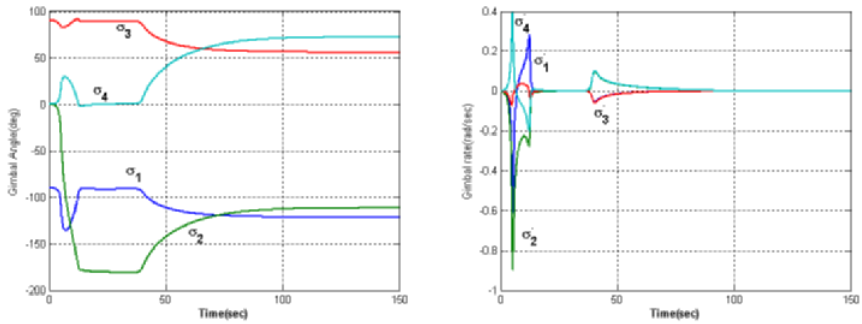
In the RDSVD steering law simulation, to prevent the gimbal rate from changing abruptly and to escape from the singular state in the very beginning, the constant values λ_1, λ_2 are set to 10^5 and 10^{-3}, respectively, and K_δ is 10^{-2} in Eqs. (27) and (28). In the simulation 1, the perturbed gain matrix is used

Table 1. Simulation 1 parameters.

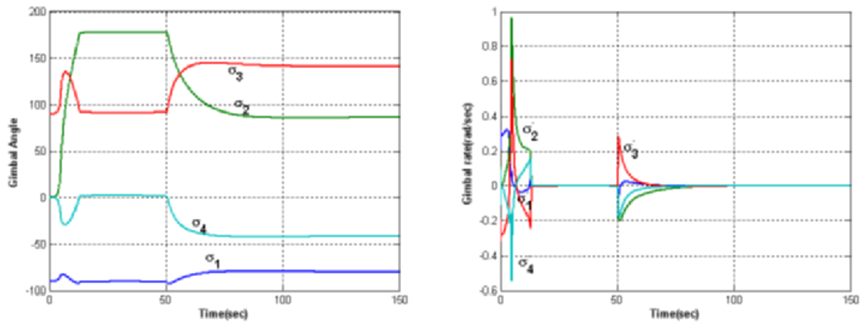
	Item	Value	Units
Initial State	ω	$[0.01\ 0\ 0]^T$	rad/sec
	β	$[0.707\ 0.707\ 0\ 0]^T$	——
	σ	$[-90, 0, 90, 0]^T$	deg
Target State	ω_f	$[0, 0, 0]^T$	rad/sec
	β_f	$[1, 0, 0, 0]^T$	——

Table 2. Simulation 2 parameters.

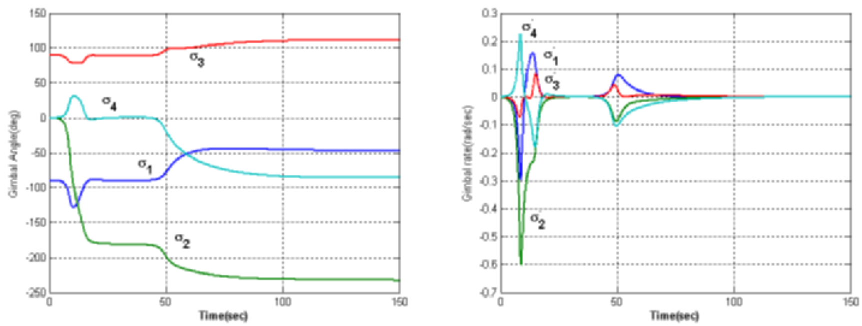
	Item	Value	Units
Initial State	ω	$[0\ 0\ 0]^T$	rad/sec
	β	$[0.707\ 0.707\ 0\ 0]^T$	——
	σ	$[0, 0, 0, 0]^T$	deg
Target State	ω_f	$[0, 0, 0]^T$	rad/sec
	β_f	$[1, 0, 0, 0]^T$	——



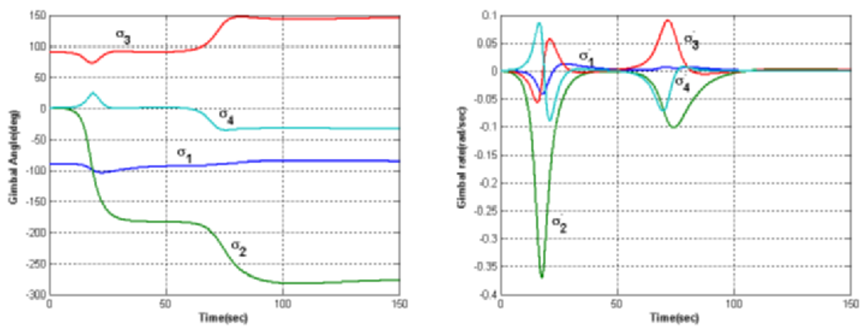
(a) SR



(b) RDSVD

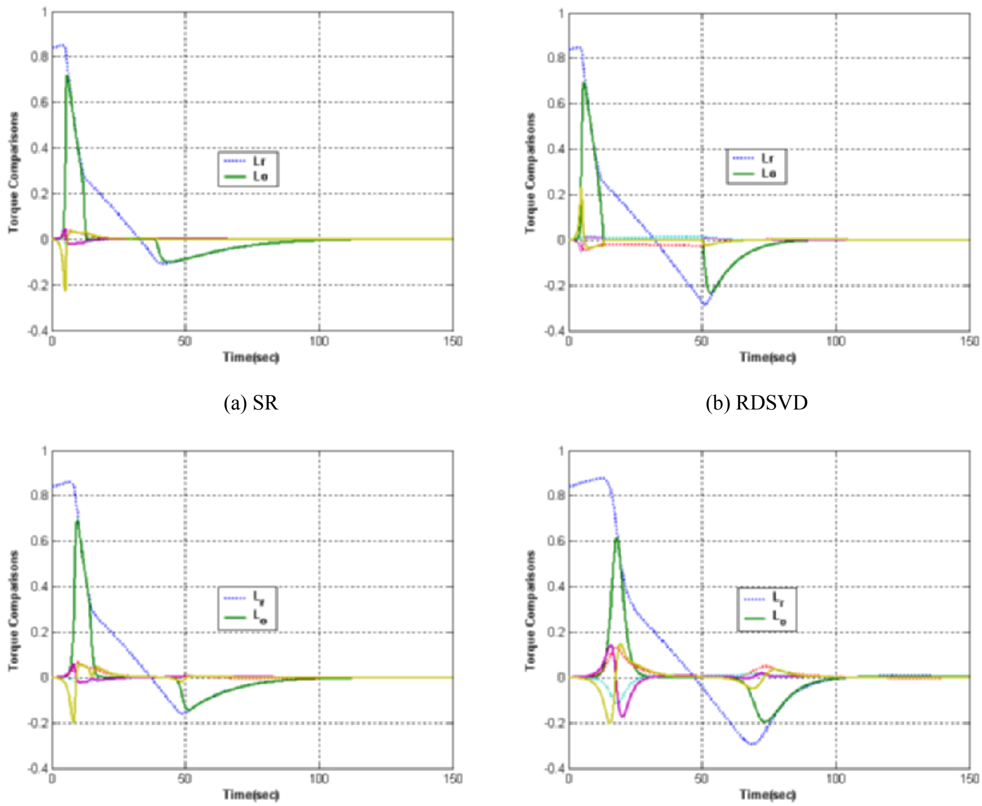


(c) RDLS-1



(d) RDLS-2

Fig. 5. Simulation 1 Results.



(a) SR (b) RDSVD

(c) RDLS-1 (d) RDLS-2

Fig. 6. Torque Comparisons.

together with feedback control laws. The perturbed gain matrix in order to prevent the required torque from being parallel to the singular direction is given by

$$K = \begin{bmatrix} K_1 & -\delta K_3 & \delta K_2 \\ \delta K_3 & K_2 - \delta K_1 \\ -\delta K_2 & \delta K_1 & K_3 \end{bmatrix} \tag{44}$$

where the perturbed gain is assigned the value δK =0.1 N·m·sec

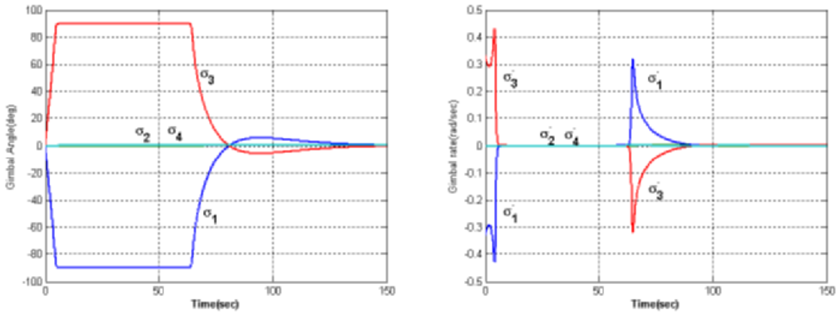
The dynamic and kinematic equations for the spacecraft with a SGCMG cluster can be written using the angular velocity ω and the Euler parameter β as

$$\dot{\omega} = -I^{-1}(\omega^{\times} I\omega + D\dot{\sigma} + \omega^{\times} D\dot{\sigma}) \tag{45}$$

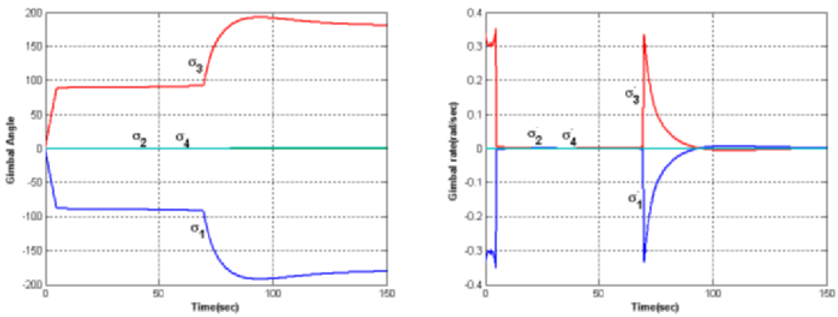
$$\dot{\beta} = \frac{1}{2} G(\beta)\omega \tag{46}$$

The following Figs. 5 and 6 reveal the results of simulation 1.

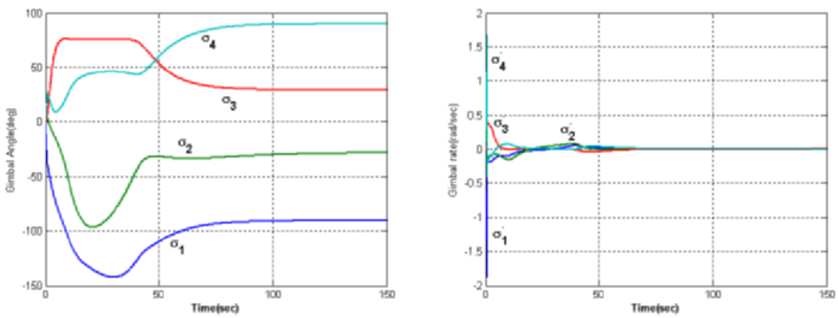
In an initial singular state, both SR and RDSVD steering laws show similar results. RDLS-1 and -2 steering laws are smoother on maneuvering compared to SR and RDSVD. All of the suggested steering laws work well in a somewhat artificial singular condition which is experienced during initial stages of maneuvering. In this simulation, the gain is perturbed for all suggested steering laws and compared with each other. However, RDSVD steering law can escape singularity by itself. In other words, the RDSVD steering law is not affected by an initial singular condition and the same results can actually be obtained by using perturbed gain matrix or not.



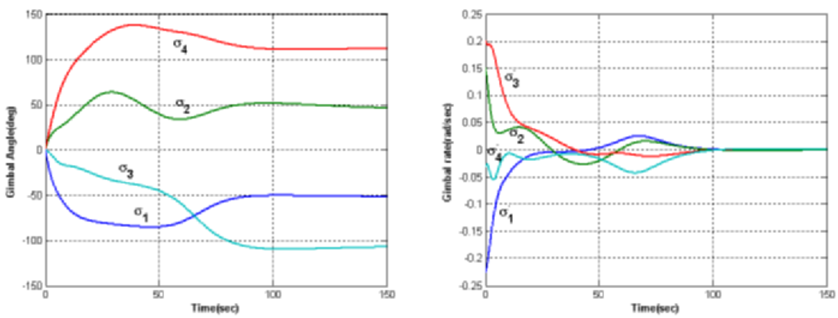
(a) SR



(b) RDSVD

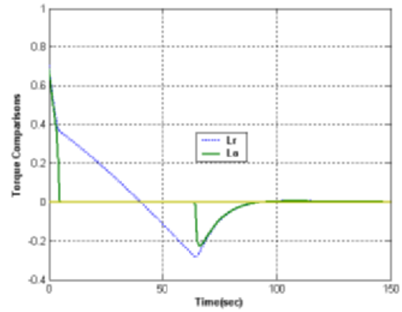


(c) RDLS-1

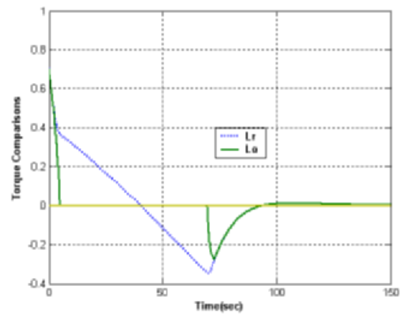


(d) RDLS-2

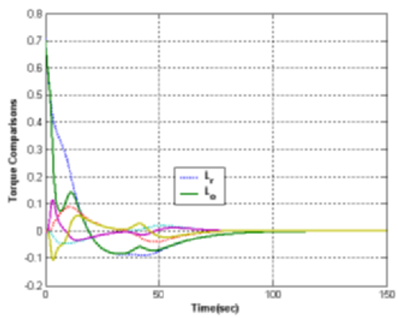
Fig. 7. Simulation 2 Results.



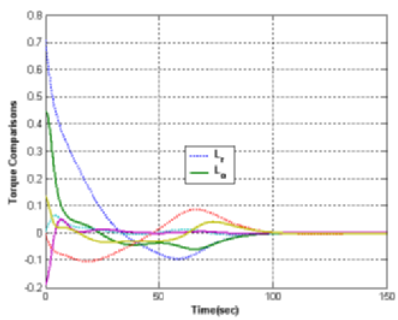
(a) SR



(b) RDSVD



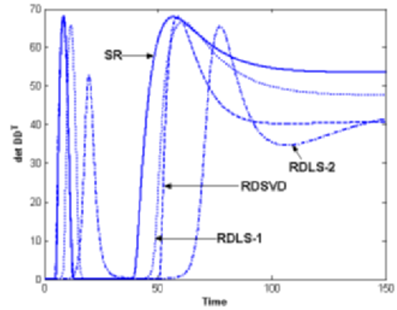
(c) RDLS-1



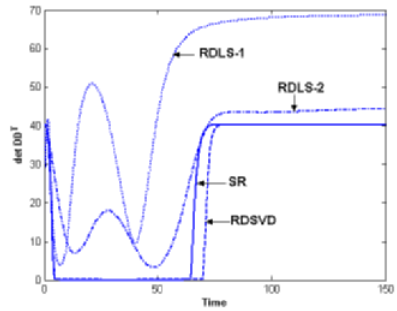
(d) RDLS-2

Fig. 8. Torque Comparisons.

The results of Simulation 2 are shown in Figs. 7 and 8. These results illustrate that RDLS steering laws



(a) Simulation1



(b) simulation 2

Fig. 9. Singularity Comparisons.

don't meet singularity and act more softly than the SR and RDSVD steering laws, as in simulation 1. The perturbed gain matrix is not used to all suggested steering laws. In Fig. 9, the det (DD^T) is shown as a measure of singularity. These results show that all suggested steering laws escaped from a singular state.

From the results of the simulations above, it is possible to state that the steering laws suggested in this paper work well compared with SR steering laws, which are well known CMG steering laws. The results of the RDSVD simulations show that the gimbal rates abruptly increases to escape from singularity, like SR steering laws. Although the output torque deviates significantly from the required torque, especially near singularity, the proposed steering law works well as the singularity-free steering law in that the gain is not perturbed. The gain in the cases of RDLS-1 and -2 is perturbed in the initial singular state condition, but these cases show smoother behavior in singularity compared with other steering laws like SR and RDSVD.

5. Conclusions

To overcome singularity problem, a new approach is presented for the design of SGCMG steering laws. The reduced dimensional singular value decomposition steering law in the present study is designed by using singular value decomposition. By the process of adopting the singular value decomposition, this new steering law eliminates one of singular values which goes to zero in a singular state. Also, by using the least square method and adopting a Jacobian matrix of reduced-dimensional forms, the reduced dimensional least square steering laws are designed. These steering laws have reduced dimensional forms that do not have a singular state. Thus, singularity-free steering laws are obtained. The difference between the methods given in this paper and those suggested previously has to do with the use of a component that may or may not cause singularity. Further, all of the suggested steering laws have been compensated four times, differences between the output torque and the required torque can be reduced in an iterative asymptotic fashion. Additionally, in RDSVD, using the newly defined switching parameter, the chattering phenomenon of gimbal rates disappears. Results of the simulations in this study show that the proposed reduced-dimensional steering laws work well, even in singularities. When the required torque is parallel to the singular directions at the beginning of the RDLS-1 and -2 steering laws simulations, it is possible to escape from the singularity by using the perturbed gain matrix. However, the RDLS-1 and -2 produce output torques more smoothly in singular states.

Acknowledgments

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References

Marqulies, G., Auburn, J. N., 1978, “Geometric Theory of Single-Gimbal Control Moment Gyro Systems,” *The Journal of the Astronautical Sciences*, Vol. 26, No. 2, pp. 159~191.

Cornick, D. E., 1979, “Singularity Avoidance Control Laws for Single Gimbal Control Moment Gyros,” *AIAA Paper*, pp. 79~1698.

Bedrossian, N. S., Paradiso, J., Bergmann, E. V., Rowell, D., 1990, “Redundant Single-Gimbal Control Moment Gyroscope Singularity Analysis,” *The Journal of Guidance, Control, Dynamics*, Vol. 13, No. 6, pp. 1096~1101.

Bedrossian, N. S., Paradiso, J., Bergmann, E. V., Rowell, D., 1990, “Steering Law Design for Redundant Single-Gimbal Control Moment Gyroscopes,” *The Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, pp. 1083~1089.

Nakamura, Y., Hanafusa, H., 1986, “Inverse Kinematic Solutions With Singularity Robustness for Robot Manipulator Control,” *The Journal of Dynamic Systems, Measurement, and Control*, Vol. 108, pp. 163~171.

Vadali, S. R., Oh, H. S., Walker, S., 1990, “Preferred Gimbal Angles for Single Gimbal Control Moment Gyroscopes,” *The Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, pp. 1090~1095.

Oh, H. S., Vadali, S. R., 1991, “Feedback control and Steering Laws for Spacecraft Using Single Gimbal Control Moment Gyros,” *The Journal of the Astronautical Sciences*, Vol. 39, No. 2, pp. 183~203.

Wie, B., Heiberg, C., Bailey, D., 2001, “Singularity Robust Steering Logic for Redundant Single-Gimbal Control Moment Gyros,” *The Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 5, pp. 271~282.

Bong, Wie., 2004, “Singularity Analysis and Visualization for Single-Gimbal Control Moment Gyros Systems,” *The Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 2, pp. 865~872.

Ford, K. A., Hall, C. D., 2000, “Singular Direction Avoidance Steering for Control-Moment Gyros,” *The Journal of the Astronautical Sciences*, Vol. 23, No. 4, pp. 648~656.

Darald, J. Hartfiel., 2000, “Matrix Theory and Applications with MATLAB,” *CRC Press*, New York, pp. 191~ 203.